PHYS 798C Spring 2024
Lecture 20 Summary

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I. FORCES ON VORTICES

After finding the force exerted by one vortex on another it is possible to calculate the force exerted on a single vortex by an externally-imposed transport current $\vec{J}$. This situation arises, for example, in a high field magnet when the field created by the current enters the superconducting wire.

Imagine an infinite line of parallel vortices and look at the net current produced by all the vortices at a point some perpendicular distance away. All of those current contributions will super-impose to create a current flowing parallel to the line. Placing a test vortex of the same vorticity there will create a net force which is given by,
$\vec{f} = \vec{J} \times \Phi_0 \hat{z}$.
We also know intuitively that the line of parallel vortices will exert a repulsive force on the test vortex, and this will directed perpendicular to the line. Again the vortex will move perpendicular to the applied current in a Lorentz force-like manner.

As the vortex moves there is a time-rate-of-change of the magnetic field at locations around the vortex core. By Faraday's law this $\partial\vec{B}/\partial t$ gives rise to an electric field as $\vec{\nabla} \times \vec{E} = -\partial\vec{B}/\partial t$. If the flux is out of the page and the current is flowing from left to right, the force exerted on the vortex is downward. As the vortex moves $-\partial\vec{B}/\partial t$ points out of the page at the instantaneous location of the vortex and $-\partial\vec{B}/\partial t$ points in to the page just below there in the direction of motion. These contributions both give an electric field pointing to the right, in the direction of the current. This will produce Ohmic losses in the normal core of the vortex, resulting in dissipation as the vortex moves. The superconductor now has a non-zero dc resistance. Finding ways to prevent the vortices from moving under such circumstances is essential to restore the zero-resistance properties of superconductors in a magnetic field. This is the subject of "vortex pinning".

Now imagine a collection of vortices being acted upon by a uniform current $\vec{J}$. If each vortex experiences the same current, it will also experience the same force, and the entire lattice will move together as a unit. As the vortices move the electric field induced is given by $\vec{E} = \vec{B} \times \vec{v}_v$, where $\vec{v}_v$ is the vortex velocity. This electric field is parallel to $\vec{J}$. Write the magnetic flux density as $|\vec{B}| = n_v \Phi_0$, where n_v is the number of vortices per unit area. Energy will be dissipated at a rate of $W = \vec{J} \cdot \vec{E}$ per unit volume.

Where does the dissipated energy go? The Bardeen-Stephen model says that it is dissipated by inducing Ohmic currents in the normal core of the vortex. The power dissipated by a single vortex is $\left(\pi a^2 L_z\right) \rho_n J^2$, where a is the radius of the vortex core, L_z is the length of the vortex in the superconductor, ρ_n is the resistivity of the electron fluid in the vortex core, often taken to be the normal state resistivity of the metal. The power dissipated in the entire sample is $\left(\pi a^2 L_z\right) \rho_n J^2 n_v A$, where A is the area of the sample. Finally, the power dissipated per unit volume of the sample is $\pi a^2 \rho_n J^2 n_v$. Equating this to $W = JE$ yields an expression for the flux-flow resistivity: $E = \rho_{ff} J$, with $\rho_{ff} = \pi a^2 \rho_n \frac{B}{\Phi_0}$. Recall that $\mu_0 H_{c2} = \frac{\Phi_0}{2\pi \xi_{GL}^2}$, so taking $a \approx \xi_{GL}$ gives $\rho_{ff} \approx \rho_n \frac{B}{B_{c2}}$. Hence the flux flow resistivity is the normal state resistivity times the fractional coverage of the sample with vortex cores. This prediction for the magneto-resistance of a superconductor is in generally good agreement with data for low-temperature superconductors, as shown on the class web site.

The flux flow resistivity can be associated with a vortex viscous force. Solving for the vortex velocity in terms of the flux flow resistivity above yields $v_v = \rho_n J / B_{c2}$. The fact that the vortex velocity scales with the applied current implies an equilibrium between a driving force and a dissipative force. We call this latter force the viscous drag force on the vortex, $\vec{f}_{drag} = -\eta \vec{v}_v$ with $\eta = \frac{J\Phi_0}{v_v}$, or $\eta = \frac{\Phi_0}{\rho_n} B_{c2}$.
The equation of motion for a single vortex acted upon by a current is then
$\vec{J} \times \Phi_0 \hat{z} - \eta \vec{v}_v + \vec{F}_{pin} = 0$, where $\vec{F}_{pin}$ is the pinning force on the vortex, which we discuss next.

II. VORTEX PINNING

Up to this point we have assumed that the superconductor is in the extreme type-II limit where $\lambda_{eff} \gg \xi_{GL}$, so that $\kappa \gg 1$. This has allowed us to ignore the vortex core compared to the energy invested in the magnetic field and super-current density distributions of the vortex. Now we go back and include the effects of the core on vortex dynamics.

Any region in space where the magnitude of the superconducting order parameter is reduced is a potential vortex pinning site. If there is a region with $|\psi| = 0$ of length D_z along the field direction, then the free energy gain of locating the vortex core there is $\Delta F_0 = \frac{\mu_0 H_c^2}{2} \pi \xi_{GL}^2 D_z$, as compared to locating in a region of space where $\psi = \psi_\infty$. Removing the vortex core from this location requires a finite energy. As long as the vortex is stationary in the presence of a current there will be no energy dissipation. This means that the superconductor has a new kind of critical current, dictated by the strength of the vortex pinning.

Pinning can be thought of (to first approximation) as creating a Hooke's law restoring force for the vortex as $\vec{F}_{pin} = -k(\vec{x} - \vec{x}_0)$, where $\vec{x}_0$ is the pinning location and $\vec{x}$ is the instantaneous location of the vortex core. The fine art of vortex pinning is to make the superconductor in to "Swiss cheese" by suppressing the order parameter in limited cylindrical regions to pin the cores, but leaving the remainder un-touched so that it can support the superconducting screening and transport currents. In this way one can have a zero resistance state in high magnetic field in the presence of a transport current, but only up to a point in either temperature, current or magnetic field.

Low-T_c superconductors can display "collective pinning" if they have a sufficiently rigid vortex lattice structure. In this case, pinning a single vortex effectively pins the entire lattice.

Under the influence of a large transport current, the Lorentz force on the vortex can exceed the pinning force, causing the vortex to de-pin. This vortex is then free to move and to dissipate energy, destroying the zero-resistance property of the superconducting state. The current at which this happens is called the de-pinning critical current, J_c^{de-pin}. In general this current density is smaller than the thermodynamic critical current density, or $J_c^{de-pin} < J_c^{GL} = H_c/\lambda_{eff}$.

One can also have the phenomenon of Thermally-Assisted Flux Flow (TAFF) in which vortices move between different pinning sites in a disordered pinning landscape due to thermally-activated motion. This is an issue with high-T_c superconductors at higher temperatures where $k_B T$ can become a substantial fraction of the pinning energies.